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Partial Wave Analysis of Nucleon–Nucleon Bethe–Salpeter Equation on the Computer*

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We present the main features of a code in the formulae manipulating language REDUCE for the evaluation of Bethe–Salpeter kernels for *NN* scattering. Our starting point is a recent work of J. J. Kubis, who gave an algorithm for the partial wave analysis of spinor Bethe–Salpeter equations in terms of the helicity formalism. Applications of these kernels apart from the evaluation of the Bethe–Salpeter equations include, for example, "matrix"–Padé approximants for partial waves.

1. INTRODUCTION

The Bethe–Salpeter equation for nucleon–nucleon scattering with single particle exchange has been considered in recent years by several authors [1, 2]. Although the equation has no solutions for physical coupling constants, it has served as a test for Padé approximants in Ref. 2. These authors found qualitative agreement in the analytic behaviour of the Padé solution as a function of the coupling constant and the exact solution as stated by Mandelstam [3].

Taking into account higher order irreducible Feynman graphs as kernels in the Bethe–Salpeter equation may lead to less singular integral equations with solutions for physical coupling constants. An even more challenging goal is, however, the calculation of Padé approximants from the perturbation series of *NN* scattering, taking into account all the graphs in each order. The work of Ref. 2 seems to encourage this direction. In that case the technique of the Bethe–Salpeter equation, especially the iteration procedure, seems to be the only adequate way of calculating Feynman graphs with a unitarity cut. Other procedures [4] have proved to be comparatively poor.

The Bethe–Salpeter equation for *NN* scattering consists of a set of eight coupled integral equations for total angular momentum $J > 0$ (four for $J = 0$). This is due to the possible couplings between states of positive and negative energies

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(in the following denoted as "ρ-spin" states) and up and down spins. The positive energy elements of the "spin matrices" of the kernel, taken on shell, give the partial wave amplitudes of the corresponding graph. Apart from being kernels in the Bethe–Salpeter integral equation, there is a further application of these matrices. It is generally believed [5] that calculating "matrix"–Padé approximants with the whole spin matrices (on shell) yields more convergent results than ordinary Padé's, as the whole matrix contains much more information than just the physical element.

The calculation of higher order Feynman graphs as kernels in the Bethe–Salpeter equation, however, is a tedious affair in view of the Dirac algebra involved. As the number of γ-matrices grows with higher order graphs, their sandwiching between Dirac spinors seems to become almost impossible in a hand calculation. In a recent work [6] J. J. Kubis has developed an algorithm to perform the partial wave analysis for spinor Bethe–Salpeter equations. It has as yet been applied to single particle exchange only, but it is applicable to the calculation of kernels from higher order irreducible Feynman graphs as well. Furthermore, this algorithm proves to be adequate for an evaluation of the Bethe–Salpeter kernels on the computer. It is the purpose of this paper to give explicit expressions for the spinors and matrix elements involved and to present the main features of a computer code for these calculations in the formulae manipulating language REDUCE, developed by A. C. Hearn [7]. We omit all symmetry properties introduced in Ref. 6, as many of them depend on the particular interaction used.

An outline of the paper is as follows: Section 2 contains a brief account of the Bethe–Salpeter equation. In Section 3 we give explicit expressions for the Dirac particle spinors for the initial and final states, following Appendix A of Ref. 6. In Section 4 we list the helicity transition amplitudes involved in the algebra. The ρ-spin analysis of Section 5 gives a survey of all single particle matrix elements to be evaluated. The actual calculation in the code is finally done by simple matrix multiplication. Section 6 contains a description of the code.

2. The Bethe–Salpeter Equation for *NN* Scattering

The Bethe–Salpeter equation is shown graphically in Fig. 1. Our kinematics are the same as in Ref. 6:

$$p_1 = (E + p_0, \mathbf{p}), \qquad p_1' = (E + p_0', \mathbf{p}'),$$
$$p_2 = (E - p_0, -\mathbf{p}), \qquad p_2' = (E - p_0', -\mathbf{p}'),$$
$$q_1 = (E + q_0, \mathbf{q}), \qquad E^2 = s/4.$$
$$q_2 = (E - q_0, -\mathbf{q}).$$

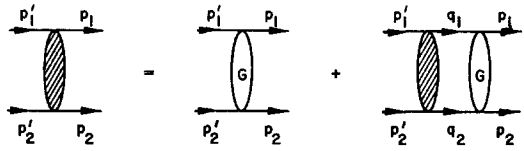

FIG. 1. Graphical representation of the Bethe–Salpeter equation.

The equation can be written symbolically as [2, 6]

$$\phi = G + GS\phi,$$

which is an operator equation for ϕ, G denoting the interaction and S the two-nucleon propagator. The conversion into an equation for partial wave amplitudes is done by sandwiching between Dirac particle states. Following Ref. 6, we write:

$$\phi = \bar{\psi}\phi\psi \qquad \text{and} \qquad G = \bar{\psi}G\psi,$$

where ψ is meant to be a two-particle Dirac wave function of given momentum, ρ-spin and helicity. The two-nucleon propagator is dependent on ρ-spin only and is given in great detail in Ref. 6. We assume that G can be written in the general form

$$G = O\left\{\sum_i \bar{\psi}(p_1)\, V_i^{(1)}\psi(q_1)\, \bar{\psi}(p_2)\, V_i^{(2)}\psi(q_2)\right\}, \tag{1}$$

where $V_i^{(1)}$ and $V_i^{(2)}$ are vertex operators for particle 1 and 2, respectively. The operator O contains the coupling constants and the propagators of the exchanged particles. In general it also stands for an integration over Feynman parameters.

It is our goal to set up a REDUCE code for the explicit evaluation of the expression in curly brackets of formula (1).

3. Explicit Form of the Dirac Particle Spinors

We write the Dirac spinors as in Appendix A of Ref. 6:

$$u_\lambda(q) = N_q \begin{bmatrix} 1 \\ \dfrac{2q\lambda}{E_q + M} \end{bmatrix} \chi_\lambda \tag{2}$$

and

$$v_\lambda(q) = N_q \begin{bmatrix} \dfrac{-2q\lambda}{E_q + M} \\ 1 \end{bmatrix} \chi_{-\lambda}, \tag{3}$$

where λ indicates the helicity ($\lambda = \pm 1/2$),

$$E_q = (q^2 + M^2)^{1/2} \qquad \text{and} \qquad N_q = ((E_q + M)/2E_q)^{1/2}.$$

These differ from the usual u's and v's [8] by the factor $(M/E_q)^{1/2}$. Positive and negative energies for the initial and final states are indicated in what follows by using large latters U and W, respectively, for their spinors. Following Ref. 6 we then have for the initial states:

$$U_{\lambda_1}(q) = u_{\lambda_1}(q), \qquad U_{\lambda_2}(q) = u_{-\lambda_2}(-q)$$

and

$$W_{\lambda_1}(q) = v_{-\lambda_1}(-q), \qquad W_{\lambda_2}(q) = v_{\lambda_2}(q).$$

The final states contain the rotated spinors

$$\chi_\lambda(\theta) = \exp(-i\sigma_2\theta/2)\,\chi_\lambda\,.$$

Using

$$\exp(-i\sigma_2\theta/2) = \cos(\theta/2) - i\sigma_2 \sin(\theta/2)$$

and

$$-i\sigma_2\chi_\lambda = (-1)^{1/2-\lambda}\chi_{-\lambda}\,,$$

we finally obtain:

$$U_{\lambda_1}(p) = \cos(\theta/2)\, u_{\lambda_1}(p) + (-1)^{1/2-\lambda_1} \sin(\theta/2)\, u_{-\lambda_1}(-p), \tag{4}$$

$$U_{\lambda_2}(p) = \cos(\theta/2)\, u_{-\lambda_2}(-p) + (-1)^{1/2+\lambda_2} \sin(\theta/2)\, u_{\lambda_2}(p) \tag{5}$$

for positive energies and

$$W_{\lambda_1}(p) = \cos(\theta/2)\, v_{-\lambda_1}(-p) + (-1)^{1/2-\lambda_1} \sin(\theta/2)\, v_{\lambda_1}(p), \tag{6}$$

$$W_{\lambda_2}(p) = \cos(\theta/2)\, v_{\lambda_2}(p) + (-1)^{1/2+\lambda_2} \sin(\theta/2)\, v_{-\lambda_2}(-p) \tag{7}$$

for negative energies.

The spinors for initial and final states are thus expressed in terms of the basic spinors (2) and (3).

4. Transition Helicity Amplitudes

According to Ref. 6, Appendix B, we have to calculate the transition helicity amplitudes $\langle \lambda_1'\lambda_2' \mid \phi \mid \lambda_1\lambda_2 \rangle$, where λ stands for the initial and λ' for the final

helicity and $\phi = V^{(1)}V^{(2)}$ is now the product of the vertex operators for particle 1 and 2, respectively (see Eq. (1)).

We have

$$\langle \lambda_1'\lambda_2' \mid \phi \mid \lambda_1\lambda_2 \rangle = \langle \lambda_1' \mid V^{(1)} \mid \lambda_1 \rangle \langle \lambda_2' \mid V^{(2)} \mid \lambda_2 \rangle,$$

and according to Eq. (B.1) of Ref. 6, the sandwiching has to be performed for the following set of helicities:

$$\begin{aligned}
\phi_1 &= \langle + \mid V^{(1)} \mid + \rangle \langle + \mid V^{(2)} \mid + \rangle, & \phi_5 &= \langle + \mid V^{(1)} \mid + \rangle \langle + \mid V^{(2)} \mid - \rangle, \\
\phi_2 &= \langle + \mid V^{(1)} \mid - \rangle \langle + \mid V^{(2)} \mid - \rangle, & \phi_6 &= \langle + \mid V^{(1)} \mid - \rangle \langle + \mid V^{(2)} \mid + \rangle, \\
\phi_3 &= \langle + \mid V^{(1)} \mid + \rangle \langle - \mid V^{(2)} \mid - \rangle, & \phi_7 &= \langle + \mid V^{(1)} \mid + \rangle \langle - \mid V^{(2)} \mid + \rangle, \\
\phi_4 &= \langle + \mid V^{(1)} \mid - \rangle \langle - \mid V^{(2)} \mid + \rangle, & \phi_8 &= \langle - \mid V^{(1)} \mid + \rangle \langle + \mid V^{(2)} \mid + \rangle.
\end{aligned}$$

We observe that the knowledge of the following single particle matrix elements gives us a complete knowledge of $\phi_1, \ldots, \phi_8$:

$$F_1^{(1)} = \langle + \mid V^{(1)} \mid + \rangle, \qquad F_2^{(1)} = \langle + \mid V^{(1)} \mid - \rangle, \qquad F_3^{(1)} = \langle - \mid V^{(1)} \mid + \rangle$$

for particle 1, and

$$\begin{aligned}
F_1^{(2)} &= \langle + \mid V^{(2)} \mid + \rangle, & F_2^{(2)} &= \langle + \mid V^{(2)} \mid - \rangle, \\
F_3^{(2)} &= \langle - \mid V^{(2)} \mid + \rangle, & F_4^{(2)} &= \langle - \mid V^{(2)} \mid - \rangle
\end{aligned}$$

for particle 2.

The asymmetry of these elements with respect to particle 1 and 2 (the extra appearance of the $\langle -, - \rangle$-element for particle 2) is due to the implicit use of conservation laws on the partial wave amplitudes, which reduces the number of matrix elements that we have to compute.

5. ρ-Spin Analysis

Omitting helicity indices and using the notation of large letters U and W for positive and negative energy states, we have the following basic states in ρ-spin space[2]:

$$\begin{aligned}
\mid + \rangle &= U^{(1)}U^{(2)}, \\
\mid - \rangle &= W^{(1)}W^{(2)}, \\
\mid e \rangle &= (U^{(1)}W^{(2)} + W^{(1)}U^{(2)})/\sqrt{2}, \\
\mid o \rangle &= (U^{(1)}W^{(2)} - W^{(1)}U^{(2)})/\sqrt{2}.
\end{aligned}$$

We have to calculate the following matrix elements:

$$\langle +, + \rangle = \bar{U}^{(1)} V^{(1)} U^{(1)} \cdot \bar{U}^{(2)} V^{(2)} U^{(2)} = F^1 \cdot F^5,$$
$$\langle +, - \rangle = \bar{U}^{(1)} V^{(1)} W^{(1)} \cdot \bar{U}^{(2)} V^{(2)} W^{(2)} = F^2 \cdot F^6,$$
$$\langle -, + \rangle = \bar{W}^{(1)} V^{(1)} U^{(1)} \cdot \bar{W}^{(2)} V^{(2)} U^{(2)} = F^3 \cdot F^7,$$
$$\langle -, - \rangle = \bar{W}^{(1)} V^{(1)} W^{(1)} \cdot \bar{W}^{(2)} V^{(2)} W^{(2)} = F^4 \cdot F^8.$$

These contain already all the necessary single particle matrix elements. All other elements are simply different combinations of the $F^1,\ldots, F^8$:

$$\langle +, e \rangle = (F^1 \cdot F^6 + F^2 \cdot F^5)/\sqrt{2},$$
$$\langle +, o \rangle = (F^1 \cdot F^6 - F^2 \cdot F^5)/\sqrt{2},$$
$$\langle -, e \rangle = (F^3 \cdot F^8 + F^4 \cdot F^7)/\sqrt{2},$$
$$\langle -, o \rangle = (F^3 \cdot F^8 - F^4 \cdot F^7)/\sqrt{2},$$
$$\langle e, + \rangle = (F^1 \cdot F^7 + F^3 \cdot F^5)/\sqrt{2},$$
$$\langle e, - \rangle = (F^2 \cdot F^8 + F^4 \cdot F^6)/\sqrt{2},$$
$$\langle o, + \rangle = (F^1 \cdot F^7 - F^3 \cdot F^5)/\sqrt{2},$$
$$\langle o, - \rangle = (F^2 \cdot F^8 - F^4 \cdot F^6)/\sqrt{2}.$$

$$\langle e, e \rangle = (F^1 \cdot F^8 + F^3 \cdot F^6 + F^2 \cdot F^7 + F^4 \cdot F^5)/2,$$
$$\langle e, o \rangle = (F^1 \cdot F^8 + F^3 \cdot F^6 - F^2 \cdot F^7 - F^4 \cdot F^5)/2,$$
$$\langle o, e \rangle = (F^1 \cdot F^8 - F^3 \cdot F^6 + F^2 \cdot F^7 - F^4 \cdot F^5)/2,$$
$$\langle o, o \rangle = (F^1 \cdot F^8 - F^3 \cdot F^6 - F^2 \cdot F^7 + F^4 \cdot F^5)/2.$$

Combining the helicity and ρ-spin analysis shows which single particle matrix elements are to be evaluated. Labelling helicity indices by lower indices (see Sec. 4) and ρ-spin indices by upper indices (1–4 for particle 1 and 5–8 for particle 2, see above), we have

$$F_i^k \qquad i = 1,\ldots, 3; \qquad k = 1,\ldots, 4$$

for particle 1, and

$$F_i^k \qquad i = 1,\ldots, 4; \qquad k = 5,\ldots, 8$$

for particle 2,

which amounts to 28 single particle matrix elements, products of which give us all the desired $\phi_1,\ldots, \phi_8$.

Finally we have to form the amplitudes f_i ($i = 1,\ldots, 8$) (B.15) of Ref. 6, which are free of kinematic singularities:

$$
\begin{aligned}
f_1 &= \phi_1 - \phi_2 , \\
f_2 &= \phi_1 + \phi_2 , \\
f_3 &= \{[\phi_3/\cos^2(\theta/2)] - [\phi_4/\sin^2(\theta/2)]\}/2, \\
f_4 &= \{[\phi_3/\cos^2(\theta/2)] + [\phi_4/\sin^2(\theta/2)]\}/2, \\
f_5 &= (\phi_5 - \phi_6)/\sin\theta, \\
f_6 &= (\phi_5 + \phi_6)/\sin\theta, \\
f_7 &= -(\phi_7 - \phi_8)/\sin\theta, \\
f_8 &= -(\phi_7 + \phi_8)/\sin\theta.
\end{aligned}
$$

These give us the starting point for the partial wave projection [6], which is in general very much dependent on the specific interaction kernel.

6. Description of the Code

It cannot be the purpose of this work to present a code which would do the calculations for all possible kernels. We would have to write separate codes for the Bethe–Salpeter kernels for singlet states ($L = J$), triplet states ($L = J$) and coupled triplet states ($L = J \pm 1$) [6]. All of these would be fairly lengthy, particularly for total angular momentum $J > 0$, and would involve—apart from some basic features—almost the same algebraic manipulations. Instead we will give the representation of the γ-matrices and spinors and of the following 16 matrix elements:

$$F_i^k (i = 1, 2; k = 1,\ldots, 8)$$

in REDUCE. The representation of the spinors is complete and nothing has to be added in further calculations. The above matrix elements are sufficient for the calculation of the Bethe–Salpeter kernel for $J = 0$ states.

Figure 2 shows the representation of the γ-matrices and spinors in REDUCE. *EI* stands for the unit matrix and *K*0,..., *K*3 and *K*5 for the γ-matrices $\gamma_0 ,\ldots, \gamma_3$ and γ_5. *U* and *V* stand for spinors of positive and negative energies. The spinors for the initial states are given by (2) and (3) and have to be set up for all possible combinations of helicity ($\lambda = \pm 1/2$) and sign of momentum ($\pm q$). These latter ones are indicated by *P* and *M* for plus and minus, respectively, where the first letter refers to the helicity and the second to the momentum. Thus *UPM*, e.g., stands for $U_{\lambda_2 = -1/2}(q) = u_{+1/2}(-q)$.

The basic spinors for the final states are $\bar{u}_\lambda(p) = u_\lambda{}^+(p) \cdot \gamma_0$. In the notation of

```
COMMENT GAMMA MATRICES;
EI:=MAT((1,0,0, 0),(0,1,0, 0),( 0, 0, 1,0),( 0,0,0, 1));
K0:=MAT((1,0,0, 0),(0,1,0, 0),( 0, 0,-1,0),( 0,0,0,-1));
K1:=MAT((0,0,0, 1),(0,0,1, 0),( 0,-1, 0,0),(-1,0,0, 0));
K2:=MAT((0,0,0,-I),(0,0,I, 0),( 0, I, 0,0),(-I,0,0, 0));
K3:=MAT((0,0,1, 0),(0,0,0,-1),(-1, 0, 0,0),( 0,1,0, 0));
K5:=MAT((0,0,1, 0),(0,0,0, 1),( 1, 0, 0,0),( 0,1,0, 0));
COMMENT SPINORS FOR INITIAL STATES;
UPP :=MAT((1),(0),( EQR),(0));
UPM :=MAT((1),(0),(-EQR),(0));
UMP :=MAT((0),(1),(0),(-EQR));
UMM :=MAT((0),(1),(0),( EQR));
VPP :=MAT((0),(-EQR),(0),(1));
VPM :=MAT((0),( EQR),(0),(1));
VMP :=MAT(( EQR),(0),(1),(0));
VMM :=MAT((-EQR),(0),(1),(0));
COMMENT BASIC SPINORS FOR FINAL STATES;
UPPB:=MAT((1,0, EPR,0))*K0;
UPMB:=MAT((1,0,-EPR,0))*K0;
UMPB:=MAT((0,1,0,-EPR))*K0;
UMMB:=MAT((0,1,0, EPR))*K0;
VPPB:=MAT((0,-EPR,0,1))*K0;
VPMB:=MAT((0, EPR,0,1))*K0;
VMPB:=MAT(( EPR,0,1,0))*K0;
VMMB:=MAT((-EPR,0,1,0))*K0;
COMMENT COMPLETE SPINORS FOR FINAL STATES;
UPB1:=X*UPPB+Y*UMMB;
UMB1:=X*UMPB-Y*UPMB;
UPB2:=X*UMMB-Y*UPPB;
UMB2:=X*UPMB+Y*UMPB;
WPB1:=X*VMMB+Y*VPPB;
WMB1:=X*VPMB-Y*VMPB;
WPB2:=X*VPPB-Y*VMMB;
WMB2:=X*VMPB+Y*VPMB;
COMMENT SUBSTITUTIONS;
EQR :=Q/EQM;
EPR :=P/EPM;
NF  :=EPM*EQM/2;
END;
```

FIG. 2. Explicit representation of spinors and γ-matrices in REDUCE.

the code they differ from the spinors for the initial states by an additional index B. The complete set of final spinors (4)–(7) is eventually given by UPB_1,..., WPB_1,... (for positive and negative energies), where P and M stand for the helicity and the indices 1 and 2 refer to particle 1 and 2. x and y stand for $\cos(\theta/2)$ and $\sin(\theta/2)$.

In addition we have used the following notation: P and Q for the momenta p and q, EPM and EQM for $E_p + M$ and $E_q + M$, NF for a normalization factor.

Figure 3 shows the evaluation of the above 16 matrix elements. *Fik* in the code stands for F_i^k and $V1$ and $V2$ represent the vertex operators $V^{(1)}$ and $V^{(2)}$ of Eq. (1). The summation over i in Eq. (1) will in general be performed in a *DO* loop.

Figure 3 shows that once the matrices are set we can just multiply them by calling their names and need not make the summation over row- and column- indices explicitly.

```
F11:=UPB1*V1*UPP;
F12:=UPB1*V1*VMM;
F13:=WPB1*V1*UPP;
F14:=WPB1*V1*VMM;
F15:=UPB2*V2*UMM;
F16:=UPB2*V2*VPP;
F17:=WPB2*V2*UMM;
F18:=WPB2*V2*VPP;
F21:=UPB1*V1*UMP;
F22:=UPB1*V1*VPM;
F23:=WPB1*V1*UMP;
F24:=WPB1*V1*VPM;
F25:=UPB2*V2*UPM;
F26:=UPB2*V2*VMP;
F27:=WPB2*V2*UPM;
F28:=WPB2*V2*VMP;
```

FIG. 3. Calculation of single particle matrix elements, V_1 and V_2 representing the vertex operators of Eq. (1).

To show our normalization (which is the same as in Ref. 2), we give the calculation of the (1, 1)-element for the singlet $L = J = 0$ state:

$$PH1 := NF * F11 * F15;$$

$$PH2 := NF * F21 * F25;$$

$$\text{KERNEL}(1, 1) := PH1 - PH2;$$

Finally, the following substitutions proved to be useful:

LET $X * X = (1 + C)/2$, $Y * Y = (1 - C)/2$, $X * Y = S/2$, $S * S = 1 - C * C$;

LET $P * P = EPM * (EPM - 2 * M)$, $Q * Q = EQM * (EQM - 2 * M)$;

FOR ALL X LET $X\uparrow(-1/2) = X\uparrow(1/2)/X$;

where S and C stand for $\sin\theta$ and $\cos\theta$, respectively. $X\uparrow(-1/2)$ means $X^{-1/2}$. The last command is used in order to combine factors of $\sqrt{2}$ and 2 properly.

To exemplify the application of our code, we write down the crossed box graph

of Fig. 4 in terms of a Feynman integral. Having performed the Chisholm algebra [9] it reads:

$$G \sim \int dx_1 \cdots dx_4 \left\{[M - \gamma^{(1)} \cdot P^{(1)}][M - \gamma^{(2)} \cdot P^{(2)}] \frac{1}{\Delta^2} + \frac{1}{2} \gamma_\mu^{(1)} \gamma^{\mu(2)} \frac{1}{\Delta}\right\},$$

$$x_1 + x_2 + x_3 + x_4 = 1.$$

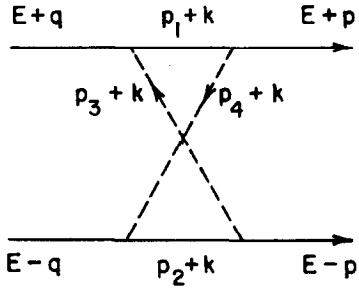

FIG. 4. Kinematics for the crossed box graph.

Here the four vectors $P^{(k)}$ are given by

$$P^{(k)} = p_k - \sum_{i=1}^{4} x_i p_i$$

and

$$\Delta = \sum_{i=1}^{4} x_i(p_i^2 - \sigma_i) - \sum_{i,j=1}^{4} x_i x_j p_i \cdot p_j ,$$

where σ_i means the square mass of particle i.

Our vertex operators are in this case

$$V^{(1)} = M - \gamma^{(1)} \cdot P^{(1)}, \qquad V^{(2)} = M - \gamma^{(2)} \cdot P^{(2)};$$

and

$$V_i^{(1)} = \gamma_i^{(1)}, \qquad V_i^{(2)} = \gamma^{i(2)} \qquad (i = 0,..., 3).$$

The vertex operators in the second term are actually the same as in the vector exchange. In Fig. 5 we present the output of our code for this latter case. We have performed the calculation for the complete box graph both by hand and computer and found that the results agree, which is an excellent check for both the hand calculation and our code. For higher order graphs, however, the hand calculation will become impossible.

```
                                          2
KERNEL(1,1) := 2*(2*EP*EK - M )

                                          2
KERNEL(1,2) := 2*(2*EP*EK + M )

                                  (1/2)
KERNEL(1,3) := 2*M*Q*2

KERNEL(1,4) := ∅

                                          2
KERNEL(2,1) := 2*(2*EP*EK + M )

                                          2
KERNEL(2,2) := 2*(2*EP*EK - M )

                                         (1/2)
KERNEL(2,3) :=  - 2*M*Q*2

KERNEL(2,4) : ∅

                                  (1/2)
KERNEL(3,1) := 2*P*M*2

                                         (1/2)
KERNEL(3,2) :=  - 2*P*M*2

KERNEL(3,3) :=  - 4*P*Q

KERNEL(3,4) := ∅

KERNEL(4,1) := ∅

KERNEL(4,2) := ∅

KERNEL(4,3) := ∅

KERNEL(4,4) := 4*EP*EK*C
```

FIG. 5. Output for $V_i^{(1)} = \gamma_i^{(1)}$, $V_i^{(2)} = \gamma^{i(2)}$ (vector exchange), where the indices 1, 2, 3 and 4 of the KERNEL stand for ${}^1S_0^+$, ${}^1S_0^-$, ${}^3P_0^e$ and ${}^3P_0^0$, respectively (see also Ref. 2).

7. CONCLUSION

The use of REDUCE for the analytic evaluation of Bethe–Salpeter kernels for *NN* scattering makes possible the evaluation of complicated higher order Feynman diagrams and makes it possible to set up higher order "matrix"-Padé approximants for partial waves. A hand calculation—apart from consuming much time—would hardly guarantee the correctness of the results. Thus our work presents a technical

step forward to the calculation of higher order Padé approximants in a field theory of strong interactions.

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